the elastic moduli at zero field and pressure are shown in Fig. 2. Data published by Rosen and Klimker [9] show similar, but significantly different, effects. The differences between Fig. 2 and Rosen's data are primarily in the variation of c_{12} ; consequently, the variations of the compressibilities β_{\parallel} and β_{\perp} with turn angle and structure, that are calculated from the Fig. 2 data, are considerably different than given by Rosen. In the present context, however, there appears no indication that magnetic ordering has any influence on the second order elastic moduli at 298°K.

The magnetic structures in Er are even more complex. The spontaneous ferromagnetic phase, with $T_c = 20^{\circ}\text{K}$, contains a spiral component. The ferromagnetic component transforms to an antiferromagnetic arrangement, between 20 and 53°K, and a modulated moment arrangement between 53 and 80°K.

The effects of the ordering on the elastic moduli are shown in Fig. 3. These effects are clearly not evident at 298°K.

The values of the second order elastic stiffness moduli at 298°K are listed in Table 2. We include here the data for Ho as given by Palmer and Lee[8]. Included in Table 2 are the values of the density[18] that are used for computing the moduli. The calculated adiabatic and isothermal bulk moduli, K_s and K_T , and also given and the last column gives the parameters $(\beta_{\parallel} - \beta_{\perp})/\beta_{V}$ calculated from isothermal values of the compressibilities. The latter parameter is that used for computing equation (3). The variations of the data with increasing atomic number (Gd -> Er) are noteworthy. The c/a ratio decreases whereas the density increases because of the so-called lanthanide contraction associated with the addition of electrons to the 4f shell. There is

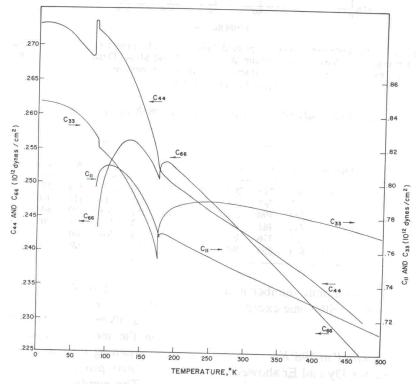


Fig. 2. Temperature dependence of the principal elastic moduli of Dy at zero magnetic field. $T_N = 179^{\circ}\text{K}$ and $T_c = 87^{\circ}\text{K}$. Absence of data for c_{11} and c_{66} at $T < 85^{\circ}\text{K}$ is result of spontaneous macroscopic distortion by anisotropic magnetostriction.

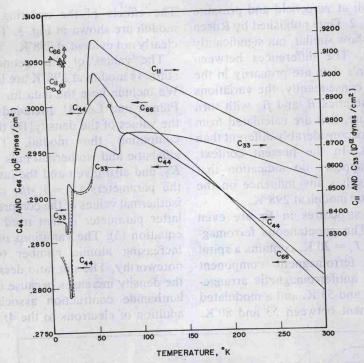


Fig. 3. Temperature dependence of principal elastic moduli in Er at zero magnetic field, where magnetic phase changes occur at ~ 20 , ~ 53 and 80° K. Dashed lines indicate thermal hysteresis in $T_c \sim 20^{\circ}$ K. Macroscopic distortion prevented measurements of c_{11} and c_{66} in spiral phase.

Table 2. Values of second order elastic moduli at 298°K of heavy rare earth metals (Kbars)

	c/a	Density	C ₁₁	c_{33}	C44	C ₆₆	C_{H}	c_{12}	c_{13}	K_s	K_T	$\left(\frac{\beta_{\parallel}-\beta_{\perp}}{\beta_{V}}\right)_{T}$
Gd Dy(F) Dy(R) Dy(P)		7.000	667	719	207	208	250	250	213	378	373	+0.007
	1·590 1·573	7.888	747	787	243	243	278	262	223	411	410	-0.005
		8·560 8·545	743	790	255	246	290	251	208	402	401	-0.009
		8.560	731	781	240	239	276	253	223	410	409	-0.015
	1 570	8.800 761		776	257	256	290	248	206	401		+0.019
Ho Er	1·570 1·569	9.064	863	855	281	279	328	305	227	455	450	+0.043

a general increase with atomic number in all of the c_{ij} and K values, with some exceptions for the Ho data.

(b) Variations of ci, with P at 298°K

Plots of the data for Dy and Er showed no significant or systematic departure from linear relations between the c_{ij} and pressure. The Gd data showed small but significant curvature

for the c_{33} vs P plot at P > 2 Kbar and for c_{44} at P = 0 and 2 Kbar. All of the data were, however, analyzed by least square statistics to obtain the most probable values for the $\mathrm{d}c_{ij}/\mathrm{d}P$, assuming no curvature. The results with indicated probable errors are given in Table 3. The similarities and differences in the effects of pressure on the c_{ij} of the three metals are evident in the pressure derivatives